4-1: More Examples

For each function and associated interval below, find any absolute maximum and absolute minimum or state that none exist.

1. $f(x) = (x^2-2x-3)^{1/3}$ on $[0,5]$

$$f'(x) = \frac{1}{3}(x^2-2x-3)^{-2/3}(2x-2) = \frac{2(x-1)}{3[(x-3)(x+1)]^{2/3}}$$

$f' = 0$ when $x=1$

f' undefined when $x=3$ or $x=-1$.

table of values

end points

critical pts in interval

x	0	5	1	3
$f(x)$	$-\sqrt[3]{3}$	$\sqrt[3]{12}$	$-\sqrt[3]{5}$	0

$25-10-3=12$ $\quad 9-6-3$

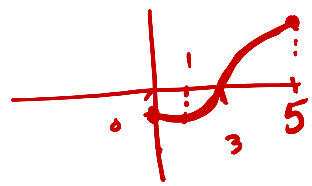
$1-2-3=-5$

answer

absolute max: $y=\sqrt[3]{12}$

absolute min: $y=-\sqrt[3]{5}$

Rough sketch



2. $g(t) = \frac{\sqrt{t}}{1+t^2}$ on $[0,3]$.

$$g'(t) = \left(\frac{(1+t^2)(\frac{1}{2}t^{-1/2}) - t^{1/2}\cdot 2t}{(1+t^2)^2}\right)\cdot\frac{2\sqrt{t}}{2\sqrt{t}}$$

$$= \frac{1+t^2-4t^2}{2\sqrt{t}(1+t^2)^2} = \frac{1-3t^2}{2\sqrt{t}(1+t^2)^2}$$

$g'=0$ when $t=\pm\sqrt{1/3}$

g' never undefined.

table of values

x	0	3	$\sqrt{1/3}$
$f(x)$	0	$\frac{\sqrt{3}}{10}$	$\frac{3^{3/4}}{4}$

aside: $\dfrac{\frac{1}{\sqrt[4]{3}}}{1+\frac{1}{3}} = \dfrac{1}{\sqrt[4]{3}}\cdot\dfrac{3}{4} =$

answer:

absolute max: $y=\frac{3^{3/4}}{4}$

absolute min: $y=0$

Rough sketch